RATIONAL POLYNOMIAL INTERPOLATION - Prof. Richard B. Goldstein

Fit the points using $$R_{m,n}(x) = \frac{P_m(x)}{Q_n(x)} = \frac{a_0 + a_1x + a_2x^2 \cdots + a_mx^m}{1 + b_1x + b_2x^2 + \cdots + b_nx^n}$$ where n = m or n = m + 1

Given: (1, 1), (4, 2), (9, 3) the solution is

$$R_{1,1}(x) = \frac{0.5454 + 0.5454x}{1 + 0.0909x} = \frac{6 + 6x}{11 + x}$$: **LINEAR**

Which can be found by solving:

$$a_0 + a_1 - b_1 = 1$$
$$a_0 + 4a_1 - 8b_1 = 2$$
$$a_0 + 9a_1 - 27b_1 = 3$$

from $a_0 + a_1x_i - b_1x_iy_i = y_i$ for $i = 1, 2, 3$

Given: (1, 1), (4, 2), (9, 3), (16, 4), (25, 5) the solution is

$$R_{2,2}(x) = \frac{0.437956 + 0.821168x + 0.0547445x^2}{1 + 0.310219x + 0.00364964x^2}$$

$$R_{2,2}(x) = \frac{34560 + 64800x + 4320x^2}{78912 + 24480x + 288x^2}$$

QUADRATIC

	Actual	linear ratio	error	quad	error	dd1	dd2
1	1.00000	1.00000	0.00000	1.00000	0.00000	0.00000	0.00000
1.5	1.22475	1.20000	0.02475	1.21672	0.00803	-0.03724	-0.02606
2	1.41421	1.38462	0.02956	1.40625	0.00796	-0.04755	-0.03103
2.5	1.58114	1.55556	0.02558	1.57534	0.00580	-0.04364	-0.02659
3	1.73205	1.71429	0.01777	1.72863	0.00343	-0.03205	-0.01823
3.5	1.87083	1.86207	0.00876	1.86938	0.00145	-0.01666	-0.00884
4	2.00000	2.00000	0.00000	2.00000	0.00000	0.00000	0.00000
4.5	2.12132	2.12903	-0.00771	2.12228	-0.00096	0.01618	0.00743
5	2.23607	2.25000	-0.01393	2.23757	-0.00150	0.03060	0.01305
5.5	2.34521	2.36364	-0.01843	2.34694	-0.00173	0.04229	0.01672
6	2.44949	2.47059	-0.02110	2.45122	-0.00173	0.05051	0.01843
6.5	2.54951	2.57143	-0.02192	2.55108	-0.00157	0.05466	0.01838
7	2.64575	2.66667	-0.02092	2.64706	-0.00131	0.05425	0.01675
7.5	2.73861	2.75676	-0.01814	2.73960	-0.00099	0.04889	0.01381
8	2.82843	2.84211	-0.01368	2.82908	-0.00065	0.03824	0.00984
8.5	2.91548	2.92308	-0.00760	2.91579	-0.00031	0.02202	0.00514
9	3.00000	3.00000	0.00000	3.00000	0.00000	0.00000	0.00000
9.5	3.08221	3.07317	0.00904	3.08193	0.00028	-0.02804	-0.00530
10	3.16228	3.14286	0.01942	3.16177	0.00051	-0.06228	-0.01049

Rational Polynomial Example – Details – Prof. Richard B. Goldstein

[1] Given points: (1, 1), (4, 2), and (9, 3) fit with $y_i = f(x_i) = \dfrac{a_0 + a_1 x_i}{1 + b_1 x_i}$

Rewrite as $y_i(1 + b_1 x_i) = a_0 + a_1 x_i$ which becomes: $a_0 + a_1 x_i - b_1 x_i y_i = y_i$

Plugging in the three points:

$$\begin{aligned} a_0 + a_1 - b_1 &= 1 \\ a_0 + 4a_1 - 8b_1 &= 2 \\ a_0 + 9a_1 - 27b_1 &= 3 \end{aligned} \quad \text{reduces to} \quad \begin{aligned} 3a_1 - 7b_1 &= 1 \\ 5a_1 - 19b_1 &= 1 \end{aligned} \quad \text{then} \quad \begin{aligned} a_1 &= 12/22 \\ b_1 &= 2/22 \end{aligned}$$

and $a_0 = 12/22$

which gives $\dfrac{\frac{12}{22} + \frac{12}{22}x}{1 + \frac{2}{22}x} = \dfrac{12 + 12x}{22 + 2x} = \dfrac{6 + 6x}{11 + x}$

[2] Given points: (1, 1), (4, 2), (9, 3), (16, 4) and (25, 5) fit with $y_i = f(x_i) = \dfrac{a_0 + a_1 x_i + a_2 x_i^2}{1 + b_1 x_i + b_2 x_i^2}$

Rewrite as $y_i(1 + b_1 x_i + b_2 x_i^2) = a_0 + a_1 x_i + a_2 x_i^2$ which becomes

$a_0 + a_1 x_i + a_2 x_i^2 - b_1 x_i y_i - b_2 x_i^2 y_i = y_i$

Plugging in the five points:

$a_0 + a_1 + a_2 - b_1 - b_2 = 1$

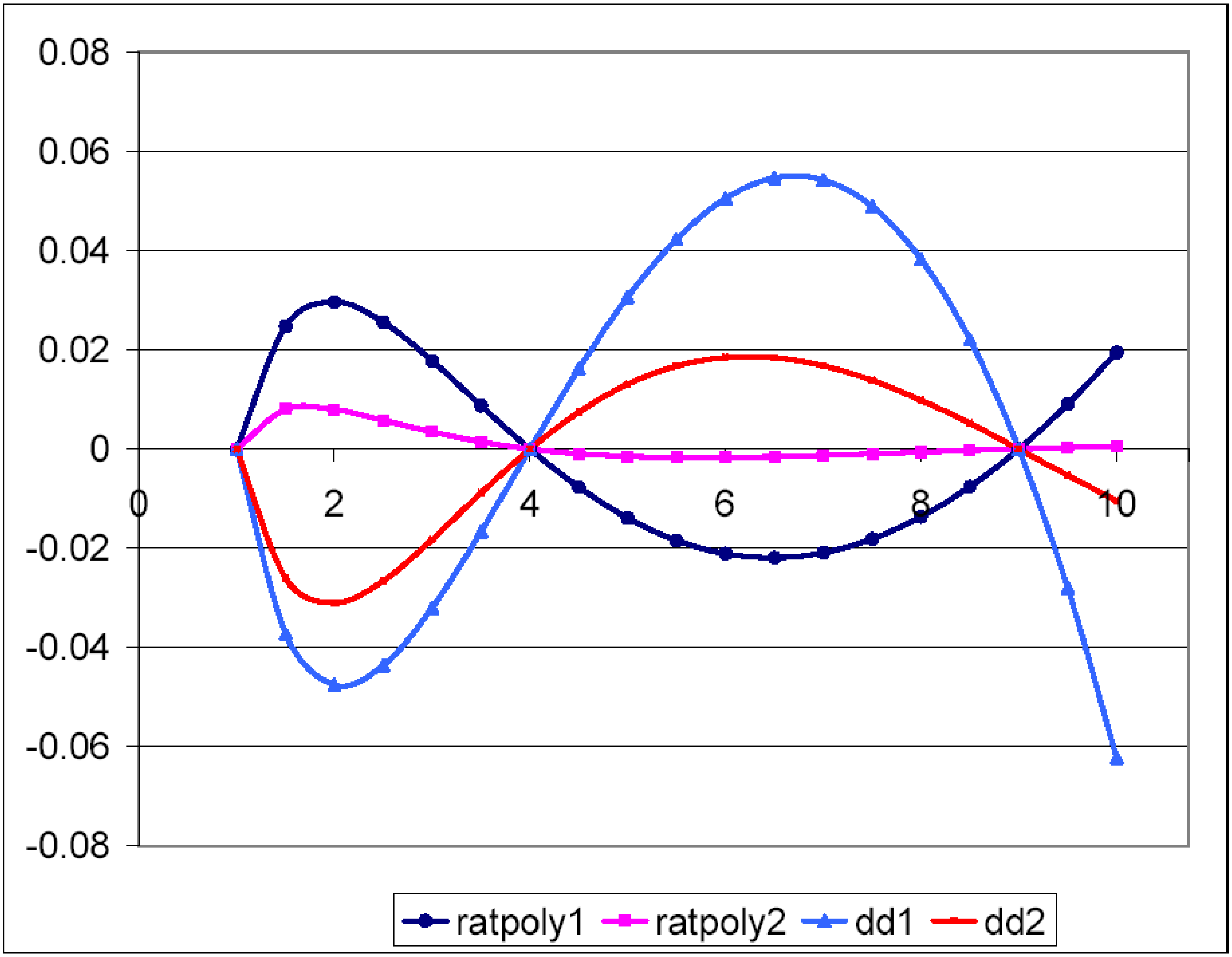
$a_0 + 4a_1 + 16a_2 - 8b_1 - 32b_2 = 2$

$a_0 + 9a_1 + 81a_2 - 27b_1 - 243b_2 = 3$

$a_0 + 16a_1 + 256a_2 - 64b_1 - 1024b_2 = 4$

$a_0 + 25a_1 + 625a_2 - 125b_1 - 3125b_2 = 5$

gives $a_0 = 0.437956$, $a_1 = 0.821168$, $a_2 = 0.0547445$, $b_1 = 0.310219$, and $b_2 = 0.00364964$


0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
2
4
6
8
10
ratpoly1
ratpoly2
dd1
dd2